

Mark Scheme (Final)

Summer 2015

Pearson Edexcel International A Level in
Statistics 3 (WST03/01)

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Summer 2015

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - B marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d...or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - \square or d... The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

June 2015
WST03 Statistics 3 Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\{w\} = 018$ or 18	018 or 18
(b)	$\{x\} = 18$	18
(c)	$\{\text{prob} =\} 0$	0
(d)	<p>Advantage: Any one of:</p> <ul style="list-style-type: none"> • <u>Simple</u> or <u>easy</u> to use also allow “quick” or “efficient” (o.e.) • It is suitable for large samples (or populations) • Gives a good spread of the data <p>Disadvantage: Any one of:</p> <ul style="list-style-type: none"> • The alphabetical list is (probably) <u>not random</u> • <u>Biased</u> since the list is not (truly) random • <u>Some combinations</u> of names are <u>not possible</u> 	<p>B1</p> <p>[1]</p> <p>B1</p> <p>[1]</p> <p>B1</p> <p>[1]</p> <p>B1</p> <p>B1</p> <p>[2]</p> <p>(Total 5)</p>
Notes		
(d)	<p>If no labels are given treat the 1st reason as an advantage and the 2nd as a disadvantage</p> <p>1st B1: for advantage</p> <p>2nd B1: for disadvantage</p> <p>“it requires a sampling frame” is 2nd B0 since the alphabetical list is given.</p> <p>Note: Do not score both B1 marks for opposing advantages and disadvantages.</p>	

Question Number	Scheme										Marks	
2. (a)		<i>A</i>	<i>B</i>	<i>C</i>	<i>L</i>	<i>N</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>Y</i>		
	Judge 1	6	3	4	9	2	8	1	5	7		
	Judge 2	8	4	5	7	3	9	1	2	6		
	or										M1	
		<i>S</i>	<i>N</i>	<i>B</i>	<i>C</i>	<i>T</i>	<i>A</i>	<i>Y</i>	<i>R</i>	<i>L</i>		
	Judge 1	1	2	3	4	5	6	7	8	9		
	Judge 2	1	3	4	5	2	8	6	9	7		
		$\sum d^2 = 4 + 1 + 1 + 4 + 1 + 1 + 0 + 9 + 1$										M1
		or $0 + 1 + 1 + 1 + 9 + 4 + 1 + 1 + 4 = 22$										$\sum d^2 = 22$ A1
		$r_s = 1 - \frac{6(22)}{9(80)}; = 0.8166666\dots$										$\frac{49}{60}$ or awrt 0.817 M1; A1
(b)	$H_0 : \rho = 0, H_1 : \rho > 0$										[5] B1	
	Critical Value = 0.7833 or CR: $r_s \geq 0.7833$										0.7833 B1	
	Since $r_s = 0.8166\dots$ it lies in the CR, or reject H_0 (o.e.)										M1	
	The two judges (or “they”) are in agreement or there is a positive correlation between the ranks of the two judges.										A1ft	
											[4] (Total 9)	
Notes												
(a)	1 st M1 for an attempt to rank at least one row (at least 4 correct)											
	2 nd M1 for an attempt at d^2 row (may be implied by sight of $\sum d^2 = 22$ or 221 for reverse ranks)											
	1 st A1 for $\sum d^2 = 22$ (or 221 if reverse ranking is used) Can be implied by correct answer.											
	3 rd M1 for use of the correct formula with their $\sum d^2$ (if it is clearly stated)											
	If the answer is not correct then a correct expression is required											
False Ranking	e.g. Alphabetic ranking: Gives Judge 1: 7 5 2 3 8 1 9 6 4 Judge 2: 7 8 5 2 3 9 4 1 6 $\sum d^2 = 162$ and $r_s = -0.35$											
	Scores: M0(for ranking), M1(for attempt at d^2 row), A0, M1 (for use of their $\sum d^2$), A0 i.e. 2 out of 5											
	Can follow through their r_s in (b)											
(b)	1 st B1 for both hypotheses stated correctly in terms of ρ (allow ρ_s) H_1 must be compatible with ranking											
	2 nd B1 for cv = 0.7833 (independent of their H_1 (no 2-tail value in tables) <u>but</u> compatible sign with their r_s)											
	M1: for a correct statement (in words) relating their r_s with their critical value.											
	e.g. “reject H_0 ”, “in critical region”, “significant”, “positive correlation”											
	May be implied by a correct contextual comment.											
 cv >1	If their cv is $ cv > 1$ (often from using normal tables) award M0A0											
	If $ their r_s > their cv $ then “significant” (o.e.) for M1 and “judges are in agreement” (o.e.) for A1ft											
	If $ their r_s < their cv $ then “not significant” (o.e.) for M1 and “judges don’t agree” (o.e.) for A1ft											
	A1ft: for a correct follow through conclusion in context.											
	“positive correlation” alone scores M1 A0											
	For reverse ranking should still say “judges are in agreement”											

Question Number	Scheme	Marks																																																				
3. (a)	$\hat{\lambda} = \frac{0(47) + 1(57) + 2(46) + 3(35) + 4(9) + 5(6)}{200} = \frac{320}{200} = 1.6$	Full exp' or at least 2 products and 320/200 seen B1 * [1]																																																				
(b)	$r = 200 \times \frac{e^{-1.6}(1.6)^2}{2!} \{= 51.68550861...\}$ $s = 200 - (40.38 + 64.61 + \text{their } r + 27.57 + 11.03) \{= 4.72449139...\}$ $r = 51.68550861... \text{ and } s = 4.72449139...$	Using $r = 200 \times \frac{e^{-1.6}(1.6)^2}{2!}$ M1 M1 A1 [3]																																																				
(c)	H_0 : Poisson (distribution) is a suitable/ sensible (model) H_1 : Poisson (distribution) is not a suitable/ sensible (model).	B1 [3]																																																				
	<table border="1"> <thead> <tr> <th>Number of accidents</th> <th>Observed</th> <th>Expected</th> <th>Combined Observed</th> <th>Combined Expected</th> <th>$\frac{(O - E)^2}{E}$</th> <th>$\frac{O^2}{E}$</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>47</td> <td>40.38</td> <td>47</td> <td>40.38</td> <td>1.0853</td> <td>54.7053</td> </tr> <tr> <td>1</td> <td>57</td> <td>64.61</td> <td>57</td> <td>64.61</td> <td>0.8963</td> <td>50.2863</td> </tr> <tr> <td>2</td> <td>46</td> <td>51.69</td> <td>46</td> <td>51.69</td> <td>0.6264</td> <td>40.9364</td> </tr> <tr> <td>3</td> <td>35</td> <td>27.57</td> <td>35</td> <td>27.57</td> <td>2.0024</td> <td>44.4324</td> </tr> <tr> <td>4</td> <td>9</td> <td>11.03</td> <td rowspan="2">15</td> <td rowspan="2">15.75</td> <td rowspan="2">0.0357</td> <td rowspan="2">14.2857</td> </tr> <tr> <td>≥ 5</td> <td>6</td> <td>4.72</td> </tr> <tr> <td colspan="5" style="text-align: right;">Totals</td> <td>4.6461</td> <td>204.6461</td> </tr> </tbody> </table>	Number of accidents	Observed	Expected	Combined Observed	Combined Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$	0	47	40.38	47	40.38	1.0853	54.7053	1	57	64.61	57	64.61	0.8963	50.2863	2	46	51.69	46	51.69	0.6264	40.9364	3	35	27.57	35	27.57	2.0024	44.4324	4	9	11.03	15	15.75	0.0357	14.2857	≥ 5	6	4.72	Totals					4.6461	204.6461	M1 [7]
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Totals					4.6461	204.6461																																																
	$X^2 = \sum \frac{(O - E)^2}{E} \text{ or } \sum \frac{O^2}{E} - 200 ; = 4.6461$ $v = 5 - 1 - 1 = 3$ $\chi^2_3(0.10) = 6.251 \Rightarrow \text{CR: } X^2 \geq 6.251$ <p>[Since $X^2 = 4.6461$ does not lie in the CR, then there is insufficient evidence to reject H_0] The number of accidents per day can be modelled by a Poisson distribution <u>or</u> the supervisor's belief is correct.</p>	M1; A1 3 6.251 B1 ft B1 ft A1 ft [7] (Total 11)																																																				
Notes																																																						
(b)	Note: Allow A1 for $s = \text{awrt } 4.74$ (found as a result of using expected values to full accuracy.)																																																					
(c)	1 st B1: for <u>both</u> hypotheses and mentioning Poisson at least once. Allow Poisson is a "good fit/model" but <u>not</u> "good method" Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion. 1 st M1: For an attempt to pool 4 accidents and ≥ 5 accidents <u>or</u> pool when $E_i < 5$ No pooling is M0 2 nd M1: For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.) 1 st A1: For awrt 4.65 (score M1M1A1 if awrt 4.65 seen) If no pooling can allow 2 nd M1 if $X^2 = 5.33$ is seen 2 nd B1ft: For $n - 1 - 1$ i.e. subtracting 2 from their n . 3 rd B1ft: For a correct ft for their $\chi^2_k(0.10)$, where $k = n - 1 - 1$ from their n . 2 nd A1ft: (Dep. on the 2nd M1) For correct comment in context based on their test statistic and their critical value that mentions accidents or supervisor . Condone mention of Po(1.6) in conclusion Score A0 for inconsistencies e.g. "significant" followed by "supervisor's belief is justified" Note: Full accuracy gives a combined expected frequency of 15.76, $\frac{(O - E)^2}{E} = 0.0366$, $\frac{O^2}{E} = 14.2766$, $X^2 = 4.64855...$ and p-value 0.199	B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling [7]																																																				

Question Number	Scheme	Marks
4. (a)	<p>Let $X =$ weight of a sack of potatoes, $X \sim N(25.6, 0.24^2)$</p> <p>So $D = X_1 - X_2 \sim N(0, 2(0.24)^2)$ or $D \sim N(0, 0.1152)$</p> <p>$\{P(D > 0.5) = \}$ $2P(D > 0.5)$</p> $= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$ $= 2 \times P(Z > 1.4731\dots) \text{ or } = 2(1 - 0.9292)$ $= 0.1416$	<p>Attempt at D and $D \sim N(0, \dots)$ $(0.24)^2 + (0.24)^2$; 0.1152 $2 \times P(D > 0.5)$ can be implied</p> <p>M1 A1; A1 dM1 dM1 awrt 0.141 or awrt 0.142</p>
(b)	<p>Let $Y =$ weight of an empty pallet, $Y \sim N(20.0, 0.32^2)$</p> <p>So $T = X_1 + X_2 + \dots + X_{30} + Y$</p> <p>$T \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$</p> <p>$T \sim N(788, 1.8304)$</p> <p>$\{P(T > 785) = \}$ $P\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$</p> $= P(Z > -2.2174\dots)$ $= 0.9868$	<p>$30(25.6) + 20$ or 788 $30(0.24)^2 + 0.32^2$ N and 1.8304 or awrt 1.83</p> <p>B1 M1 A1 M1 awrt 0.987</p>
Notes		
(a)	<p>1st M1: For clear definition of D and normal distribution with mean of 0 (Can be implied by 3rd M1)</p> <p>1st A1: for correct use of $\text{Var}(X_1 - X_2)$ formula</p> <p>2nd A1: for 0.1152</p> <p>2nd dM1: For realising need $2 \times P(D > 0.5)$ (Dependent on 1st M1 i.e. must be using suitable D)</p> <p>3rd dM1: Dep on 1st M1 for standardising with 0.5, 0 and their s.d. ($\neq 0.24$) Must lead to $P(Z > +ve)$ (o.e.) $P(Z > 1.47)$ implies 1st M1 1st A1 2nd A1 and 3rd M1 Correct answer only will score 6 out of 6</p>	
(b)	<p>B1: For a mean of $30(25.6) + 20$. Can be implied by 788.</p> <p>1st M1: For $30(0.24)^2 + 0.32^2$. Can be implied by 1.8304 or awrt 1.83 Allow M1 for swapping error i.e. $30 \times 0.32^2 + 0.24^2$ if the expression is seen</p> <p>1st A1: For normal and correct variance of 1.8304 or awrt 1.83. Normality may be implied by standardisation</p> <p>2nd M1: For standardising with 785 with their mean and st. dev. ($\neq 0.24$) Must lead to $P(Z > -ve)$ oe.</p> <p>2nd A1: awrt 0.987 Correct answer only will score 5 out of 5</p> <p>Note: Calculator answers are (a) 0.14071..., (b) 0.98670...</p>	

[6]

[5]

(Total 11)

Question Number	Scheme	Marks																																
5.	H_0 : Grades and gender are independent (or not associated) H_1 : Grades and gender are dependent (or associated)	“grades” and “gender” mentioned at least once.	B1 (1)																															
	<table border="1"> <thead> <tr> <th>Observed</th> <th>Male</th> <th>Female</th> </tr> </thead> <tbody> <tr> <td>Distinction</td> <td>37</td> <td>44</td> </tr> <tr> <td>Merit</td> <td>127</td> <td>96</td> </tr> <tr> <td>Unsatisfactory</td> <td>36</td> <td>20</td> </tr> </tbody> </table>	Observed	Male	Female	Distinction	37	44	Merit	127	96	Unsatisfactory	36	20	An attempt to convert percentages to observed frequencies.	M1																			
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			All observed frequencies are correct.	A1																														
	<table border="1"> <thead> <tr> <th>Expected</th> <th>Male</th> <th>Female</th> <th>Totals</th> </tr> </thead> <tbody> <tr> <td>Distinction</td> <td>45</td> <td>36</td> <td>81</td> </tr> <tr> <td>Merit</td> <td>123.889</td> <td>99.111</td> <td>223</td> </tr> <tr> <td>Unsatisfactory</td> <td>31.111</td> <td>24.889</td> <td>56</td> </tr> <tr> <td>Totals</td> <td>200</td> <td>160</td> <td>360</td> </tr> </tbody> </table>	Expected	Male	Female	Totals	Distinction	45	36	81	Merit	123.889	99.111	223	Unsatisfactory	31.111	24.889	56	Totals	200	160	360	Some attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$	M1											
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		Can be implied by a correct E_i	A1																															
		All expected frequencies are correct to nearest integer.	A1																															
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Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$																															
37	45	1.422	30.422																															
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Totals		5.104	365.104																															
		Accept 2 sf accuracy for the M1 mark.	A1																															
		All correct $\frac{(O - E)^2}{E}$ or $\frac{O^2}{E}$ terms to either 2 dp or better.	A1																															
		Allow truncation. (\Rightarrow by awrt 5.1 if 3 rd M1 seen)	A1																															
	$X^2 = \sum \frac{(O - E)^2}{E}$ or $\sum \frac{O^2}{E} - 360$;= awrt 5.1	awrt 5.1	A1 (7)																															
	$\nu = (3 - 1)(2 - 1) = 2$	$(\nu =) 2$ (Can be implied by 5.991)	B1																															
	$\chi^2_2(0.05) = 5.991 \Rightarrow \text{CR: } X^2 \geq 5.991$	For 5.991 only	B1																															
	Since $X^2 = 5.1$ does not lie in the CR, then there is insufficient evidence to reject H_0		M1																															
	Business Studies <u>grades</u> and <u>gender</u> are independent <u>or</u> There is no association between Business Studies <u>grades</u> and <u>gender</u> . <u>Or</u> <u>Head of department's</u> (belief) is correct		A1ft (4)																															
			[12] (Total 12)																															
	Notes																																	
	Final M1: For a correct statement linking their test statistic and their critical value (> 3.8) Note: Contradictory statements score M0. E.g. “significant, do not reject H_0 ”.																																	
	Final A1ft: For a correct ft statement in context – must mention “grades” and “gender” or “sex” <u>or</u> “head of department” Condone “relationship” or “connection” here but not “correlation”. e.g. “There is no evidence of a relationship between grades and gender”																																	
5.10 only	Just seeing 5.10... only can imply 1 st 3 Ms but loses 1 st 3 As so can score 4 out of 7 (Qu says “show..”)																																	
	Note: Full accuracy gives $X^2 = 5.104356...$ and p-value 0.0779																																	

Question Number	Scheme	Marks																															
5.	<p>Mark Scheme for candidates who use percentages instead of observed values.</p> <p>H_0 : Grades and gender are independent (or not associated) “grades” and “gender” mentioned at least once.</p> <p>H_1 : Grades and gender are dependent (or associated)</p>	B1																															
	<table border="1"> <thead> <tr> <th>Observed</th> <th>Male</th> <th>Female</th> </tr> </thead> <tbody> <tr> <td>Distinction</td> <td>18.5</td> <td>27.5</td> </tr> <tr> <td>Merit</td> <td>63.5</td> <td>60.0</td> </tr> <tr> <td>Unsatisfactory</td> <td>18.0</td> <td>12.5</td> </tr> </tbody> </table>	Observed	Male	Female	Distinction	18.5	27.5	Merit	63.5	60.0	Unsatisfactory	18.0	12.5	These marks cannot be obtained. M0 A0																			
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$X^2 = \sum \frac{(O - E)^2}{E} \text{ or } \sum \frac{O^2}{E} - 360 ; = 2.8518$	This mark cannot be obtained. A0																																
$\nu = (3 - 1)(2 - 1) = 2$	This mark cannot be obtained. A0																																
$\chi^2_2(0.05) = 5.991 \Rightarrow \text{CR: } X^2 \geq 5.991$	($\nu =$) 2 (Can be implied by 5.991) B1																																
<p>Since $X^2 = 2.86$ does not lie in the CR, then there is insufficient evidence to reject H_0</p>	For 5.991 only B1																																
	Not available since comes from incorrect working. A0																																
	[12] (Total 12)																																
Notes																																	
If a candidate uses percentages rather than observed values then they can obtain a maximum of 6 marks . They can get B1 M0A0 M1A0 M1A0A0 B1B1M1A0.																																	

Question Number	Scheme	Marks
6. (a)	$\left\{ \hat{\mu} = \frac{\sum x}{n} = \frac{1570}{50} = \right\} \bar{x} = 31.4$ $\left\{ \hat{\sigma}^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \right\} s_x^2 = \frac{49467.58 - 50(31.4)^2}{50-1}$ $= 3.460816...$	$\bar{x} = \mathbf{31.4}$ B1 cao M1 A1ft awrt 3.46 [4]
(b)	[Let $Y =$ time taken to complete obstacle course in the afternoon.] $H_0: \mu_x = \mu_y$, $H_1: \mu_x > \mu_y$ $(z =) \frac{31.4 - 30.9}{\sqrt{\frac{3.46}{50} + \frac{3.03}{50}}}$ $= 1.38781...$ CR: $Z \geq 1.6449$ or probability = awrt 0.082 or awrt 0.083 Since $z = 1.38781...$ does not lie in the CR, then there is insufficient evidence to reject H_0 Conclude that the <u>mean time</u> to complete the obstacle course is the same for the early <u>morning</u> and late <u>afternoon</u> .	B1 M1 A1ft awrt 1.39 1.6449 or better B1 M1 A1 [7]
(c)	\bar{X} and \bar{Y} are both approx. normally distributed or $\bar{X} - \bar{Y}$ normal (Condone \bar{x} and \bar{y})	B1 [1]
(d)	Have assumed $s^2 \approx \sigma^2$ or variance of sample \approx variance of population	B1 [1]
Notes		
(a)	B1: 31.4 cao Allow 31 minutes, 24 seconds. 1 st M1: A correct expression for either s or s^2 (ignore label) 1 st A1ft: A correct expression for s^2 with their ft \bar{x} . 3 rd A1: awrt 3.46 (Correct answer scores 3 out of 3)	
(b)	1 st B1: Both hypotheses stated correctly, with some indication of which μ is which. Eg: μ_M, μ_A 1 st M1: For an attempt at $\frac{a-b}{\sqrt{\frac{c}{50} + \frac{d}{50}}}$ with at least 3 of a, b, c or d correct. Allow \pm 1 st A1ft: for $\pm \frac{\text{their } 31.4 - 30.9}{\sqrt{\frac{\text{their } 3.46}{50} + \frac{3.03}{50}}}$ Allow $D = \bar{x} - \bar{y}$ $1.64 \sim 1.65 = \frac{D-0}{\sqrt{\frac{3.46}{50} + \frac{3.03}{50}}}$ [SE = 0.360277..] 2 nd A1: for awrt 1.39 (possibly \pm) (Allow for CV $D =$ awrt 0.593) (NB $d = 0.5$) Correct answer scores M1A1ftA1 <u>but</u> $0 - (31.4 - 30.9) \rightarrow -1.39$ loses this 2 nd A mark 2 nd B1: Critical value of 1.6449 or better (seen). Allow for probability = awrt 0.082 or awrt 0.083 Note: p-values are 0.0823 (tables) and 0.0826 (calculator). 2 nd M1: For a correct statement linking their test statistic and their critical value. Note: Contradictory statements score M0. E.g. "significant, do not reject H_0 ". 3 rd A1: For a correct statement in context that accepts H_0 (no ft) Condone "no difference in mean times" Must mention " <u>mean time</u> ", " <u>morning</u> " and " <u>afternoon</u> " or " <u>both times of day</u> "	
(c)	B1 E.g. $\bar{X} \sim N(\dots)$ need both. Allow in words e.g "sample means are normally distributed"	
(d)	B1 condone only mentioning "x" or "y" <u>but</u> watch out for $s_x = s_y$ or $\sigma_x = \sigma_y$ which scores B0	

Question Number	Scheme	Marks
7.	Let $X =$ score on a die	
(a)	$E(S) = 3.5, \text{Var}(S) = \frac{35}{12}$	$E(S) = 3.5$ B1 $\text{Var}(S) = \frac{35}{12}$ or awrt 2.92 B1
(b)	$\text{So, } \bar{S} \sim N\left(3.5, \frac{\frac{35}{12}}{40}\right)$ or $\bar{S} \sim N\left(3.5, \frac{7}{96}\right)$	B1ft [2]
	$P(\bar{S} < 3) = P\left(Z < \frac{3 - 3.5}{\sqrt{\frac{7}{96}}}\right) \{= P(Z < -1.85164\dots)\}$	M1
	$\{= 1 - 0.9678\} = 0.0322$	0.032 to 0.0322 A1
		[3] (Total 5)
Notes		
(a)	2 nd B1 allow awrt 2.92	
(b)	$\text{B1ft for } \bar{S} \sim N\left(3.5, \frac{\frac{35}{12}}{40}\right)$ seen or implied. Follow through their $E(S)$ and their $\text{Var}(S)$	
	NB $\frac{7}{96} = 0.07291\dot{6}$ accept awrt 0.0729	
	M1 for an attempt to standardise with 3, their mean (>3) and $\sqrt{\frac{\text{their Var}(S)}{40}}$. Must lead to $P(Z < -ve)$	
	A1 for 0.032 ~ 0.0322	
ALT ES	B1ft for $\sum S \sim N\left(140, \frac{350}{3}\right)$ where 140 is $40 \times$ their $E(S)$ and variance is $40 \times$ their $\text{Var}(S)$	
	M1 for $P\left(Z < \frac{120 - 140}{\sqrt{\frac{350}{3}}}\right)$ or $P\left(Z < \frac{119.5 - 140}{\sqrt{\frac{350}{3}}}\right) \{= P(Z < -1.8979\dots)\}$	
	A1 for 0.032~0.0322 or (with continuity correction) 0.0287 (tables) or 0.0289 (calculator).	

Question Number	Scheme	Marks	
8. (a)	$\left\{ \bar{x} = \frac{29.74 + 31.86}{2} \right\} \Rightarrow \bar{x} = 30.8$ $\text{"1.96"} \left(\frac{\sigma}{\sqrt{n}} \right) = 31.86 - 30.8 \quad \text{or} \quad 2(\text{"1.96"}) \left(\frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74$ $SE_{\bar{x}} = \frac{31.86 - 30.8}{1.96} = 0.540816... = 0.54 \text{ (2dp)}$	$\bar{x} = 30.8$ This can be implied. See note. awrt 0.54	B1 M1 A1 [3]
(b)	A 90% CI for μ is $\bar{x} \pm 1.6449 \left(\frac{\sigma}{\sqrt{n}} \right)$ $= 30.8 \pm 1.6449(0.54)$ $= (29.91, 31.69)$	$(\text{their } \bar{x}) \pm (\text{their } z)(\text{their } SE_{\bar{x}} \text{ from (a)})$ (awrt 29.9 , awrt 31.7)	B1 M1 A1 [3]
(c)	Let X = number of confidence intervals containing μ or Y = number of confidence intervals not containing μ So $X \sim \text{Bin}(4, 0.9)$ or $Y \sim \text{Bin}(4, 0.1)$ $P(X \geq 3)$ or $P(Y \leq 1) = {}^4C_3(0.9)^3(0.1) + (0.9)^4$ $= 0.2916 + 0.6561 = 0.9477$	${}^4C_3(0.9)^3(0.1) + (0.9)^4$ 0.9477 or 0.948	M1 A1 oe A1 [3]
Notes			(Total 9)
(a)	B1: $\bar{x} = 30.8$ may be implied by $1.96 \left(\frac{\sigma}{\sqrt{n}} \right) = [31.86 - 30.8] = 1.06$ <u>or</u> $2(1.96) \left(\frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74$ M1: A correct equation for either a width or a half-width involving a z -value $1.5 \leq z \leq 2$ Eg: "their z " $\left(\frac{\sigma}{\sqrt{n}} \right) = 31.86 - \text{"30.8"}$ ft their \bar{x} <u>or</u> $2(\text{"their } z") \left(\frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74$ or "their z " $(SE_{\bar{x}}) = 31.86 - \text{"30.8"}$ <u>or</u> $2(\text{"their } z")(SE_{\bar{x}}) = 31.86 - 29.74$ are fine for M1.	NB $\frac{53}{98}$ as final answer is A0 Condone $\bar{x} \pm 1.96\sigma = \dots$ for B1 and M1 but A0 even if they say " σ = standard error = 0.54" Otherwise answer only of 0.54 scores 3 out of 3	
(b)	B1 for use of 1.6449 or better in an attempt at a CI formula. Need at least 1.6449 (their SE) M1 for attempt at CI ft their values and provided $1 \leq z \leq 1.7$		
(c)	M1: States or applies either $X \sim \text{Bin}(4, 0.9)$ <u>or</u> $Y \sim \text{Bin}(4, 0.1)$ Condone M1 for $0.9^4 + 0.9^3 \times 0.1$ (o.e.) 1 st A1: ${}^4C_3(0.9)^3(0.1) + (0.9)^4$ or $(0.9)^4 + {}^4C_1(0.1)(0.9)^3$ oe 2 nd A1: 0.9477 or 0.948		

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